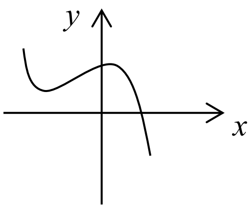
EDEXCEL ADVANCED EXTENSION AWARD (9801) – JUNE 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme		Marks
1.	$\sin 5x + \sin x \equiv 2\cos 2x \sin 3x$	use of one	M1
	$\cos 5x + \cos x \equiv 2\cos 2x \cos 3x$	both correct	A1
	Equation becomes: $0 = 2\cos 2x(\sin 3x - \cos 3x)$		
	i.e. $\cos 2x = 0$		M1
	$\tan 3x = 1$		M1
	$\cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$	1st solution	M1
	$\tan 3x = 1 \Rightarrow 3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \ldots \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \ldots$	1st solution	M1
		a correct 2nd solution	A1
	$\therefore \quad x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$	all 4	A1
			(8 marks)
2.	$(1 - 4x)^p =$		
	$\frac{p(p-1)}{2!}(-4x)^2 + \frac{p(p-1)(p-2)}{3!}(-4x)^3 + \frac{p(p-1)(p-2)(p-3)}{4!}(-4x)^4 \ldots$		M1 at least x^2 term
	Equation: $\frac{p(p-1)}{2!} \times 4^2 = \frac{p(p-1)(p-2)(p-3)}{4!} \times 4^4$	attempt equation	M1 (ignore x's)
	$1 = \frac{(p-2)(p-3)\times 16}{12}$	cancel or factor $p(p-1)$	M1
	i.e. $0 = 4p^2 - 20p + 21$		A1
	i.e. $0 = (2p-3)(2p-7)$	solving	M1
	i.e. $p = \frac{3}{2}$ or $\frac{7}{2}$	both	A1
	coefficient of $x^3 > 0 \Rightarrow p(p-1)(p-2) < 0$	x^3 coefficient examined	M1
	so $p \neq 0$ and $p \neq 1$		A1
	$p \neq \frac{7}{2} \quad \therefore p = \frac{3}{2}$		A1
			(9 marks)

Question Number	Scheme	Marks
3.	At (14, 1), $t = 1$	B1
	$\frac{dy}{dx} = \frac{-4t}{15-3t^2}$, at $t = 1$ $\frac{dy}{dx} = -\frac{1}{3}$ $\therefore$ gradient of normal, = 3	M1, M1, A1
	Equation of normal is: $y - 1 = 3(x - 14)$ [or $y = 3x - 41$]	M1
	Cuts C when: $3 - 2t^2 - 1 = 3(15t - t^3 - 14)$	M1
	i.e. $3t^3 - 2t^2 - 45t + 44 = 0$ simplified cubic = 0	A1
	$(t - 1)$ is a factor $(t - 1)$ is a factor	M1
	$\therefore$ $(t - 1)[3t^2 + t - 44] = 0$ [ ]	A1
	i.e. $(t - 1)(3t - 11)(t + 4) = 0$	M1
	$t = \frac{11}{3}, -4$	
		(11 marks)
4.	$\frac{d}{dx}: 3x^2 + 3y^2\frac{dy}{dx}, -3x\frac{dy}{dx} - 3y = 0$	M1 A1, M1
	$\frac{dy}{dx} = 0, \Rightarrow y = x^2$	M1, A1
	substitute back: $x^3 + x^6 - 3x \times x^2 = 48$	
	i.e. $x^6 - 2x^3 - 48 = 0$	A1
	$(x^3 - 8)(x^3 + 6) = 0$	M1
	so $x^3 = 8 \Rightarrow x = 2$ and $y = 4$	A1
	or $x^3 = -6 \Rightarrow x = -6^{\frac{1}{3}}$ and $y = 6^{\frac{2}{3}}$	A1 (10)
	$\frac{d}{dx}$ again: $6x + 6y\left(\frac{dy}{dx}\right)^2 + 3y^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 3x\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$	M1
	$(x = 2, y = 4) < 0$ $\therefore (2, 4)$ is maximum	A1
	$\frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{2x}{x - y^2}$ check signs of y''	M1
	$(x = -6^{\frac{1}{3}}, y = 6^{\frac{2}{3}}) > 0$ $\therefore (-6^{\frac{1}{3}}, 6^{\frac{2}{3}})$ is minimum	A1 (4)
		(14 marks)
Alternative for last 4 marks	 Sketch from top left to bottom right	M1
	Min in 2nd quad, max in first quad	M1
	$\therefore$ minimum at $(-6^{\frac{1}{3}}, 6^{\frac{2}{3}})$	A1
	maximum at (2, 4)	A1 (4)

Question Number		Scheme	Marks
5.	(*a*)	$\sin(\cos x) = 0 \Rightarrow \cos x = 0$ (, ...) (ignore others)	M1
		$\therefore\ x = \frac{\pi}{2}$ or $-\frac{\pi}{2}$ so A is $(\frac{\pi}{2}, 0)$, C is $(-\frac{\pi}{2}, 0)$	A1 both
		$x = 0 \Rightarrow y = \sin(1)$, i.e. B is $(0, \sin(1))$	B1 (3)
	(*b*)	$\frac{dy}{dx} = -\cos(\cos x)\sin x$	M1
		$x = 0$ at B and $\left(\frac{dy}{dx}\right)_0 = 0$, $\therefore$ B is a stationary point	A1 (2)
	(*c*)	For $\theta \geq 0$, $\sin\theta \leq \theta$	
		$\cos x \geq 0$ for $x \in [0, \pi/2] \Rightarrow \sin(\cos x) \leq \cos x$; equality when $x = \frac{\pi}{2}$	A1 cso; B1
		Equation of BC is $y = \frac{-\sin(1)}{\pi/2}x + \sin(1)$ attempt equation BC	M1
		$\therefore$ convex line is below curve, $\therefore$ $\sin(1)\left[1 - \frac{2}{\pi}x\right] \leq \sin(\cos x)$	A1 cso
		equality when $x = 0, \frac{\pi}{2}$	B1 (both) (6)
	(*d*)	$\int_0^{\frac{\pi}{2}} \cos x \ dx = \left[\sin \ x\right]_0^{\frac{\pi}{2}}, = 1$ $\therefore I < 1$	M1, A1 cso
		$\int_0^{\frac{\pi}{2}} \sin(1)\left[1 - \frac{2}{\pi}x\right]dx$ **OR** area of triangle $= \frac{1}{2} \times \frac{\pi}{2}\sin(1)$, $\therefore I > \frac{\pi}{4}\sin(1)$	M1, A1 cso (4)
			(15 marks)

cso = correct solution only

Question Number	Scheme	Marks
6. (*a*)	$(3, 0) \Rightarrow m_1 = 3^{n_1}$, $m_2 = 3^{n_2}$ and $m_2 > m_1$ $\therefore\ n_2 > n_1$	M1
	curves are symmetric and n_1, n_2 are *both* even	M1
(*b*)	so $n_1 + n_2 = 12 \Rightarrow n_1 = 2,\ n_2 = 10$; or $n_1 = 4,\ n_2 = 8$	A1; A1 (4) (−1 each extra solution)
	Area = $2\int_0^3 \left([m_2 - m_1] + [x^{n_1} - x^{n_2}]\right)\mathrm{d}x$	M1
	$= 2\left[(m_2 - m_1)x + \dfrac{x^{n_1+1}}{n_1+1} - \dfrac{x^{n_2+1}}{n_2+1}\right]_0^3$	M1 A1 ft
	smallest area when m_1 and m_2 are closest	M1
	i.e. $n_1 = 4$, $n_2 = 8$, $m_1 = 3^4$, $m_2 = 3^8$	A1
	$= 2\left[3(3^8 - 3^4) + \dfrac{3^5}{5} - \dfrac{3^9}{9} - 0\right]$ Use of correct limits	M1
	$= 2 \times 3^9 - 2 \times 3^5 + \frac{2}{5} \times 3^5 - 2 \times 3^7$ combine powers o3 that differ by 1 or less	M1
	$= 2\,[3^9 - 3^7 - \frac{8}{5} \times 3^5]$ OR $16 \times 3^7 - \frac{8}{5} \times 3^5$ OR $\frac{712}{5} \times 3^5$	A1 (8)
(*c*)	Gradient same $\Rightarrow -n_1x^{n_1-1} = -n_2x^{n_2-1}$ Equation based on d*y*/d*x*	M1
	i.e. $\dfrac{n_1}{n_2} = x^{n_2-n_1}$ OR $x = (n_2 - n_1)\sqrt{\dfrac{n_1}{n_2}}$ single *x*	M1
	$n_1 = 4,\ n_2 = 8 \Rightarrow x^4 = \frac{1}{2}$ or $x = \sqrt[4]{\frac{1}{2}}$	
	$n_1 = 2,\ n_2 = 10 \Rightarrow x^8 = \frac{1}{5}$ or $x = \sqrt[8]{\frac{1}{5}}$ both cases	A1 ft
	$(\frac{1}{2})^2 > \frac{1}{5}$ or $\sqrt[8]{\frac{1}{5}} = \sqrt[4]{\frac{1}{\sqrt{5}}}$ and $\frac{1}{\sqrt{5}} < \frac{1}{2}$, $\therefore$ greatest $x = \left(\frac{1}{2}\right)^{\frac{1}{4}}$	M1, A1 (5)
		(17 marks)

ft = follow-through mark

Question Number		Scheme		Marks
7.	(*a*)	$pq = \frac{1}{2} \not\Rightarrow p = \frac{1}{2}$ or $q = \frac{1}{2}$ (line 3)	identify; explain	B1; B1 (2)
	(*b*)	$x^3 + \frac{3}{4}x - \frac{1}{2} = 0 \Rightarrow (x - \frac{1}{2})(x^2 + \frac{1}{2}x + 1) = 0$	attempt to divide	M1
			correct quadratic	A1
		i.e. $x = \frac{1}{2}$ or $x^2 + \frac{1}{2}x + 1 = 0$, discriminant $= (\frac{1}{2})^2 - 4$		M1
		$< 0 \quad \therefore$ no real roots (so only root is $x = \frac{1}{2}$)		A1 cso (4)
	(*c*)	$x = \alpha$ is a root $\Rightarrow \alpha^3 + \beta\alpha - \alpha = 0$, i.e. $\beta = 1 - \alpha^2 \quad (\alpha \neq 0)$		M1, A1
		$x^3 + \beta x - \alpha \equiv (x - \alpha)[x^2 + \alpha x + 1]$		M1 [A1]
		Discriminant of $x^2 + \alpha x + 1$ is $\alpha^2 - 4$		M1
		$\therefore x = \alpha$ is the only real root if $\alpha^2 - 4 < 0$, i.e. $\lvert\alpha\rvert < 2$ (✱)		A1 cso (6)
	(*d*)	Student's method: $x(x^2 + \beta) = \alpha$		
		$\Rightarrow x = \alpha$ or $x^2 + \beta = \alpha$		M1
		require $\alpha - \beta > 0$		
		$\alpha^2 + \alpha - 1 > 0$		
		cvs $\alpha = \dfrac{-1 \pm \sqrt{5}}{2}$	attempt cvs	M1
			2 correct cvs	A1
		$\therefore \dfrac{\sqrt{5} - 1}{2} < \alpha < 2$ or $-2 < \alpha < -\dfrac{\sqrt{5} - 1}{2}$		A1, A1 (7)
				(19 marks)
		STYLE INSIGHT & REASONING		
	(*a*)	**S marks**		
		For a novel or neat solution to any of questions 3—7. Apply once per question in up to 3 questions		
		S2 if solution is fully correct in principle and accuracy		S6 (S2 × 3)
		S1 if principle is sound but includes a minor algebraic or numerical slip		
		T mark		
		For a good and largely accurate attempt at the whole paper		T1
				(7 marks)

(✱) indicates final line given in the question paper; ft = follow-through mark; cso = correct solution only